Enriched Lenses

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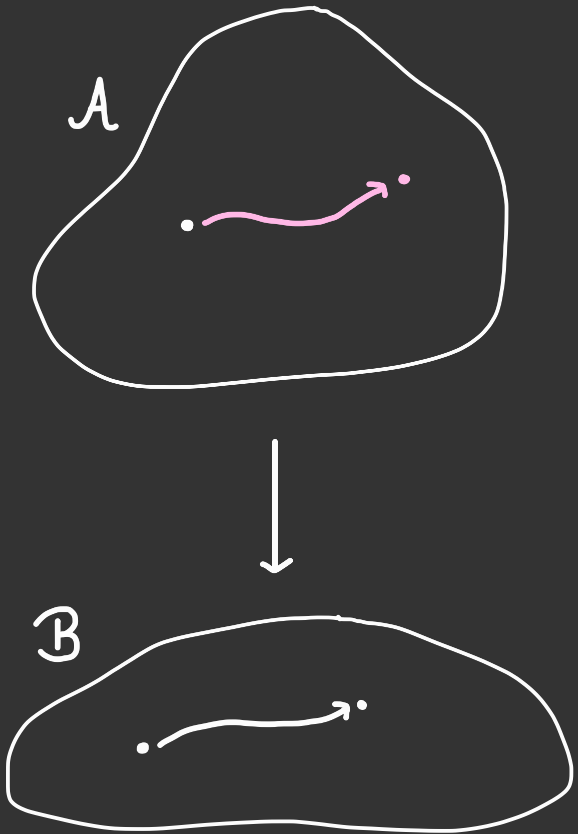
OVERVIEW & MOTIVATION

Lenses model bidirectional transformations:

- object assignment ⇝ consistency relation
- functor ⇝ forwards consistency restoration
- cofunctor ⇝ backwards consistency restoration



MOTIVATING QUESTION

What is the correct way to restore consistency if the systems are enriched?

1. Introduce enriched cofunctors & lenses.
2. Share some examples, incl. weighted lenses.

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ENRICHED FUNCTORS VS. COFUNCTORS

Assumption: enrichment in a distributive monoidal category $\mathcal{V}$

enriched functor $F : \mathcal{A} \to \mathcal{B}$

$$\mathrm{obj}(\mathcal{A}) \xrightarrow{F} \mathrm{obj}(\mathcal{B})$$

$$\sum_{x \in X} \mathcal{A}(a,x) \xrightarrow{[F_{a,x}]} \mathcal{B}(Fa,b)$$

$X = F^{-1}\{b\}$

+ axioms

enriched cofunctor $(F,\varphi) : \mathcal{A} \to \mathcal{B}$

$$\mathrm{obj}(\mathcal{A}) \xrightarrow{F} \mathrm{obj}(\mathcal{B})$$

$$\mathcal{B}(Fa,b) \xrightarrow{\varphi_{a,b}} \sum_{x \in X} \mathcal{A}(a,x)$$

$X = F^{-1}\{b\}$

+ axioms

$\mathcal{V} = \omega\mathrm{Set}$ weighted functor

$v \in \mathcal{A}(a,a')$ $\quad |F_{a,a'}v| \leqslant |v|$

$\mathcal{V} = \omega\mathrm{Set}$ weighted cofunctor

$u \in \mathcal{B}(Fa,b)$ $\quad |\varphi_{a,b}u| \leqslant |u|$

ENRICHED COFUNCTORS AS SPANS

Assumption: enrichment in a distributive monoidal & extensive category $\mathcal{V}$

enriched cofunctor $(F,\varphi): \mathcal{A} \to \mathcal{B}$ $\simeq$ span of enriched functors:

$$\mathcal{A} \xleftarrow{P} \mathcal{C} \xrightarrow{Q} \mathcal{B}$$

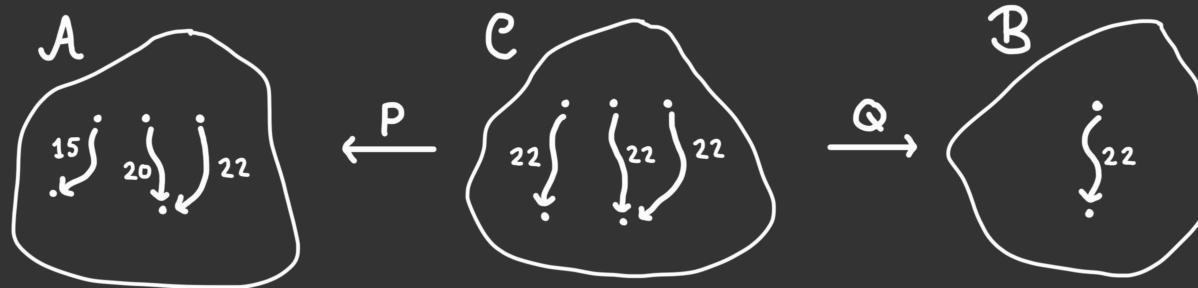
$P : \mathrm{obj}(\mathcal{C}) \to \mathrm{obj}(\mathcal{A})$ invertible

$[Q_{c,x}] : \sum_{x \in X} \mathcal{C}(a,x) \to \mathcal{B}(Qa,b)$ invertible

$\mathcal{V} = \omega\mathrm{Set}$

weighted cofunctor

$|\varphi_{a,b}\, u| \leq |u|$



ENRICHED LENSES

enriched functor $F : \mathcal{A} \to \mathcal{B}$

$$F_{a,a'} : \mathcal{A}(a,a') \longrightarrow \mathcal{B}(Fa, Fa')$$

$$F : \mathrm{obj}(\mathcal{A}) \longrightarrow \mathrm{obj}(\mathcal{B})$$

$$\varphi_{a,b} : \mathcal{B}(Fa, b) \longrightarrow \sum_{x \in F^{-1}\{b\}} \mathcal{A}(a,x)$$

enriched cofunctor $(F, \varphi) : \mathcal{A} \longrightarrow \mathcal{B}$

enriched lens $(F, \varphi) : \mathcal{A} \nrightarrow \mathcal{B}$

$$\begin{tikzcd}
\mathcal{B}(Fa, b) \arrow[d, "\varphi_{a,b}"] \arrow[dd, bend right=60, "\mathrm{id}"'] \\
\sum_{x \in F^{-1}\{b\}} \mathcal{A}(a,x) \arrow[d, "{[F_{a,x}]}"] \\
\mathcal{B}(Fa, b)
\end{tikzcd}$$

$\mathcal{V} = \omega\mathrm{Set}$ **weighted lens**

$u \in \mathcal{B}(Fa, b) \qquad |\varphi_{a,b}\, u| = |u|$

EXAMPLES OF ENRICHED LENSES

$\mathcal{V}$	Coproduct	lifting map	notes
$(\mathrm{Set}, \times, 1)$	$\bigsqcup$	$\mathcal{B}(Fa,b) \xrightarrow{\varphi_{a,b}} \bigsqcup_{x \in F^{-1}\{b\}} \mathcal{A}(a,x)$	delta lens Diskin et al. 2011
$(\omega\mathrm{Set}, +, 0)$	$\bigsqcup$	same, s.t. $\lvert\varphi u\rvert = \lvert u\rvert \;\; \forall u$	weighted lens Perrone 2021
$([0,\infty], +, 0)$	inf	$d(Fa,b) \geqslant \inf_{x \in F^{-1}\{b\}} d(a,x)$	submetry is an example
$(\mathrm{Ab}, \otimes, \mathbb{Z})$	$\bigoplus$ biproduct	$\{\mathcal{B}(Fa,b) \xrightarrow{\varphi_{a,b}} \mathcal{A}(a,x)\}_{x \in F^{-1}\{b\}}$	additive lens
$\mathrm{Fam}(\mathcal{V})$	$\bigsqcup$	similiar to first example	any monoidal category $\mathcal{V}$

WHY IS THIS NOTION "CORRECT"?

$\mathbb{D} \rightsquigarrow \mathbb{M}\mathrm{nd}_{\mathrm{ret}}(\mathbb{D}) \rightsquigarrow \mathbb{\Gamma}(\mathbb{M}\mathrm{nd}_{\mathrm{ret}}(\mathbb{D}))$

double category with companions	monads, monad morphisms, & monad retromorphisms	right-connected completion $\rightsquigarrow$ lenses between monads
$\mathbb{M}\mathrm{at}(\mathcal{V})$	enriched categories, functors, & cofunctors	enriched lenses
$\mathbb{S}\mathrm{pan}(\mathcal{E})$	internal categories, functors, & cofunctors	internal lenses
$\mathbb{F}\text{-}\mathbb{R}\mathrm{el}$	topological spaces, continuous maps, & open maps	open continuous maps

SUMMARY & FUTURE WORK

$\boxed{\text{enriched lens}} \rightsquigarrow \boxed{\text{vertical arrow in } \mathbb{\Gamma}(\mathbb{M}\mathrm{nd}_{ret}(\mathbb{M}\mathrm{at}(\mathcal{V})))}$

$\mathcal{V} = \mathrm{Set}$ $\mathcal{V} = [0,\infty]$

$\mathcal{V} = \omega\mathrm{Set}$ $\mathcal{V} = \mathrm{Ab}$

=

$\boxed{\text{enriched functor}}$

+

$\boxed{\text{enriched cofunctor}} \simeq \boxed{\text{span of enriched functors}}$

$\mathcal{V}$ distributive monoidal

$\mathcal{V}$ extensive

More examples!

- $\mathcal{V} = \mathrm{Poly} \rightsquigarrow$ collectives
- $\mathcal{V} = \mathcal{P}M$, M monoid, $\mathcal{P}$ powerset

More theory!

- Can we better understand enriched split opfibrations using enriched lenses?
- Grothendieck construction for weighted lenses? Enriched?
- 2-cells of enriched cofunctors.